Abstract

A unified measure of inequality, volatility, and mobility risk is developed from well-known decompositions of a generalized entropy inequality measure. Decomposition by population subgroup is applied to panel data measuring income across people over time, where subgroups are taken to be individual people, so “within-group” inequality is a measure of the variability of individual income over time and “between-group” inequality is a measure of inequality in long-run mean incomes. The variability of individual income over time is further decomposed into “volatility” and “mobility risk” using individual-specific trends in income. I apply the decompositions to several decades of U.S. data and find every component (inequality, volatility, and mobility risk) increasing over time, and a large impact of taxes. I further find large swings in the progressivity of income growth after taxes that are not observed in pretax income, consistent with the known tax regimes in recent U.S. history.
**Introduction**

There has been a renewed interest in recent years in income inequality, but also economic mobility (both moving up and moving down), and income volatility, the year-to-year variations in income that families may or may not be able to smooth over. This is not just about the business cycle: the percentage of Americans worried about being laid off nearly quadrupled from 12 to 46 percent\(^1\) from 1982 to 1998, as the economy improved dramatically. Over the same time frame, workers reported their subjective probability of job loss fell from close to 20 to less than 10 percent.\(^2\)

To address these concerns, researchers, journalists, and politicians have joined the fray, seeking to measure and explain income inequality, mobility and volatility. These related phenomena have different implications; as Senator Schumer said, “If you’re holding a job but your share of the pie is getting smaller, that’s a different set of policy needs than if you keep losing your job” (quoted by Leonhardt 2007). To date, however, there has been no unified approach to measuring these phenomena.

I define an aggregate measure of income risk as half the squared coefficient of variation (or the general entropy measure with parameter 2, denoted GE\(_2\)) of incomes measured over both people and time. The aggregate measure can be decomposed into an inequality component measuring dispersion in mean incomes, a volatility component measuring the average dispersion of fluctuations about person-specific trends, and a mobility component measuring the dispersion of person-specific trends. I then apply this decomposition to Panel Study of Income Dynamics (PSID) data from the United States to characterize trends in inequality, volatility, and mobility over the last several decades. I also examine the regressivity of income growth in these data.

**Background**

It is of little use to discuss inequality without some mention of changing incomes over time. They may be changing due to short-lived transitory shocks, or more permanent changes, but either kind of change induces greater volatility in the income stream and greater relative\(^3\) mobility. Some view these changes as mitigating inequality (frequently citing Schumpeter 1955 or Friedman 1962), but if these changes reflect income risk, they lower well-being, holding constant the mean level of income. This paper inclines toward the latter viewpoint, characterizing observed changes in income as reflecting dispersion, and I define an aggregate measure of income risk that can be decomposed into a long-run inequality component, a volatility component, and a mobility component.

Of course, greater absolute mobility, especially growth in real incomes, may change our interpretation of these other features. A doubling of real incomes may make us less

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\(^1\) Nichols and Zimmerman (2008), quoting poll results from the International Survey Research Corporation in Chicago; see also Greenspan (1997).

\(^2\) Leonhardt (2007), quoting Gallup poll results.

\(^3\) I am calling growth in incomes relative to average growth “relative mobility” so both volatility and mobility may result in reranking of individuals within the income distribution in any one period or over time.
worried about increasing trends in volatility combined with constant relative mobility and increasing inequality, though the change in measured (scale-invariant) inequality due to a doubling of incomes is nil. I will briefly discuss income growth toward the end, but for most of the text, I will restrict my attention to total mobility, or the sum of absolute and relative mobility. I will focus exclusively on measuring inequality and volatility and mobility, not characterizing their welfare consequences.  

Inequality in observed incomes is not inequality in well-being, or important outcomes such as mortality rates. Even if we regard income as a valid measure of well-being, inequality of observed incomes is a poor measure of inequality of income distributions. Measured inequality is positive when all incomes are drawn from the same distribution. That is, if every individual in society has income in every period that is a random draw from the same distribution (implying equality of opportunity) the inequality of observed outcomes across individuals will be nonzero, and overstate inequality of opportunity.

On the other hand, inequality estimates in survey data are typically biased downward. Breunig (2001) shows that the bias of the GE2 estimator (estimating half the squared coefficient of variation, or the general entropy measure with parameter 2), used in the rest of this paper, has the sign of three times the coefficient of variation (CV) less twice the population skewness. So for income distributions that exhibit large positive skew, the bias of the GE2 estimator is usually negative. This property also holds if we imagine having population data on income and estimating an inequality parameter for the superpopulation, or family of populations from which the current population data are drawn. So if the inequality of incomes in the hypothetical population or superpopulation is positive, we will typically underestimate that positive level of inequality.

Thus there may be two offsetting biases: variation in observed outcomes may overstate variation in potential outcomes, but measured variation in observed outcomes may understate potential variation in observed outcomes. In other words, population inequality (of outcomes) may overstate inequality of opportunity, but sample inequality may understate population inequality. However, these biases are unavoidable, and there is good reason to think they are small given a large sample over many years.

Volatility is also never, strictly speaking, observed. As the volatility of a stock is estimated using historical data on changes in price, so income volatility is often measured as variation in income over time. However, this reflects behavioral changes, measurement error, and both short-term and long-term real changes in income. Some authors attempt to decompose variability of income over time into permanent and transitory shocks, but this requires specifying a model of income dynamics that applies to all individuals (see, e.g., Lillard and Weiss 1978; Moffitt and Gottschalk 1995; Baker 1997), and it is very likely that no such model would survive empirical tests of its restrictions in a more flexible

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4 See, for example, Atkinson (1970) and Gottschalk and Spolaore (2002) for connections to social welfare. I will follow the lead of Sen (1973) in pursuing descriptive measures.

5 Many authors have pointed out that well-being is multidimensional and cannot be characterized using a simple scalar variable like observed income; see, for example, Atkinson and Bourguignon (1982), Maasoumi (1986), and Bourguignon and Chakravarty (2003).

model that nests it (for example by treating additional implications of the model not strictly required to identify parameters as overidentifying restrictions in a Generalized Method of Moments framework). In short, income exhibits individual heterogeneity in levels and growth rates which are not independent of the history of income levels and gains. In contrast, the approach adopted in this paper imposes no distributional assumptions on income at a point in time or on income growth (though linear and loglinear trends are measured, these are conceived as short-run approximations to arbitrary individual-specific paths of income over time).

Mobility has been defined in many different ways, and the term encompasses many different concepts. Relative mobility and absolute mobility have already been discussed, and various authors define various versions of directional mobility or exchange mobility. One measure of mobility (Shorrocks 1978a; Fields 2007) depends on the reduction in inequality due to averaging or summing individual incomes across time. This definition implicitly assumes that the sum or average of income is the key factor in determining well-being, and that the effects of variation in income around the mean have negligible welfare consequences. Other definitions of mobility rely on transition matrices between states (Shorrocks 1978b; Geweke et al. 1986; Alcalde-Unzu et al. 2006), but these founder on several well-known difficulties associated with transition matrices. Using transition matrices, definitions of categories characterizing states will affect results (e.g., quintiles or deciles), and the current state is typically not a sufficient statistic for transition probabilities as is required for a Markov process (meaning that the transition matrix for any given pair of periods does not fully characterize the process).

Methods

Inequality and volatility are always characterized as some measure of dispersion, or variability, of a distribution. The overarching idea of the method is that we want to measure variability in income using panel data, observations both across individuals and across time. Imagine measuring income of three individuals a, b, and c for three years 1, 2, and 3: we can arrange observations first by time:

| a1 b1 c1 | a2 b2 c2 | a3 b3 c3 |

or first by individual:

| a1 a2 a3 | b1 b2 b3 | c1 c2 c3 |

which suggests two decompositions by group, where group is defined by a time index t or alternatively by an individual index i. The natural choice (Shorrocks 1984) of inequality measure for decompositions by group is the generalized entropy measure GE2, equal to half the squared coefficient of variation.

Suppose we observe L people, indexed by i running from 1 to L, observed at T points in time, for \( N = LT \) observations on income \( y \). Consider first a decomposition by

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7 Fields and Ok (1999) offer a review of this broad field of research. Benabou and Ok (2001) define desirable mobility as progressive income growth and Mazumder (2008) attempts to measure only upward mobility.

8 Another choice of analysis unit is of course possible, for example family or household, but these are much less convenient when dealing with panel data, since composition may change over time.
population subgroup following Shorrocks (1984) where the population is all person-years and subgroups are people:

\[
GE_2 = \frac{1}{2\bar{y}^2} \left[ L^{-1} \sum_{i=1}^{L} T^{-1} \sum_{t=1}^{T} (\bar{y}_{it})^2 - \bar{y}^2 \right] + \frac{1}{2\bar{y}^2} \left[ L^{-1} \sum_{i=1}^{L} T^{-1} \sum_{t=1}^{T} y_{it}^2 - (\bar{y}_i)^2 \right] = B_i + W_i
\]

where \( \bar{y}_i \) indicates the within-person sample mean of income over all time periods observed, and \( \bar{y} \) is the sample mean of income over all persons and time periods. The first term \( B_i \) represents variation across individuals in their mean income over some time period of \( T \) years, i.e. \( T \)-year inequality. The second term \( W_i \) represents variation of individual income around mean income, i.e. volatility.9

We can think of the term \( B_i \) as measuring the variance of fixed effects from a regression (of income on no explanatory variables). The variance estimate in the term \( B_i \) overstates the true variance of fixed effects (Aaronson et al. 2007), because of the sampling variation in the sample mean. That is, let \( \mu_i \) be the deviation of each person’s mean income from the population mean income; then because

\[
\mathbb{E}[\hat{\mu}_i^2] = \mu_i^2 + \mathbb{E}[(\hat{\mu}_i - \mu_i)^2]
\]

the variation in individual-specific sample means (over some time frame) overstates the variation in individual-specific true means by the amount of sampling variation in the sample mean, and we would like to estimate the sampling variation and subtract it from our variance estimate.

The sampling frame one imagines here is not the usual one assumed that guarantees the favorable properties of a simple random sample, where we have independent observations on individual income at a point in time. For measuring inequality in mean incomes over some time period we would like incomes to be measured without error; if we had repeated independent measurements of income at each point in time, we could calculate an estimate of the sampling variation at each point in time, and thereby adjust our estimate of the variance mean downward by the mean squared standard error.

In practice however we have variation across time that reflects both sampling variation in the long-run mean of income and changes in true income over that period of time. The sampling variation in the individual-specific sample means, computed using

Alternatively, consider a decomposition of the form

\[
GE_2 = \frac{1}{2\bar{y}^2} \left[ L^{-1} \sum_{i=1}^{L} T^{-1} \sum_{t=1}^{T} y_{it}^2 - (\bar{y}_i)^2 \right] + \frac{1}{2\bar{y}^2} \left[ L^{-1} \sum_{i=1}^{L} T^{-1} \sum_{t=1}^{T} (\bar{y}_t)^2 - (\bar{y})^2 \right] = W_i + B_t
\]

where the first term \( W_i \) represents variation around a “year effect” and measures variation across people from year-specific means, or a weighted average of year-specific inequality measures. The second term \( B_t \) measures variations in individual year means around the longer-run mean, or “aggregate volatility” in individual incomes. The first term in each of these expressions represents variation across individuals at a point in time, or inequality. The second term represents variation across time periods, or volatility. Only the first decomposition relates to our conception of volatility of individual incomes, however. Therefore we will focus only on the decomposition of \( G \) into \( W_i \) and \( B_t \).
variation in incomes across time, therefore incorporates two sources of variation, one due to sampling variation in the mean and one due to changing incomes (mobility and volatility). We can regard using as our estimate of sampling variation the square of the standard error of the mean computed in the usual way, dividing the sample standard deviation by the number of time periods, as a likely upper-bound estimate. Thus unadjusted estimates of variance in mean incomes and those adjusted downward to account for the effect of sampling variation on our estimate of variation plausibly bound variation in true means (both versions are presented in Results below).

The sampling variation in the individual-specific sample means is estimated by the individual-specific sample standard deviation over the number of time periods, or the summand of $W_i$ divided by $(T - 1)$. So the variance estimate in the term $B_i$ should be reduced by that amount; in order to keep the total unchanged, we must add the same quantity to $W_i$ like so:

$$GE_2 = \left( B_i - \frac{1}{2T^2} \left[ L^{-1} \sum_{i=1}^{L} \frac{\sum_{t=1}^{T} (y_{it} - \overline{y}_i)^2}{T(T-1)} \right] \right) + \left( W_i + \frac{1}{2T^2} \left[ L^{-1} \sum_{i=1}^{L} \frac{\sum_{t=1}^{T} (y_{it} - \overline{y}_i)^2}{T(T-1)} \right] \right)$$

Combining terms in the second component, we can see that the summand in the second component is now scaled by $T$ over $(T - 1)$, which is the usual adjustment for variance estimation. In other words, the revised decomposition can be written

$$GE_2 = \left( B_i - \frac{W_i}{T-1} \right) + \left( \frac{T}{T-1} \right) W_i = I + D$$

Thus both estimates are corrected for sampling variation, and the sum remains unchanged. We call the corrected components $I$ for inequality in mean income and $D$ for deviations around the mean. To be precise, these components are inequality in individual-specific mean incomes over time (for $I$) and variance of deviations over time around the individual-specific means (for $D$).

We can further decompose the second term $D$ ("deviations") into a component due to individual trends in income, and a component due to variations around trend. This is most intuitively understood by imagining regressing individual income on a time trend and a constant, and letting the sum of squared residuals be defined as the component due to variations around trend. The difference between the second term $D$ and the mean over individuals of the individual-specific sum of squared residuals (or variation in detrended and demeaned income) is the individual-specific variance of predicted income over $T$ years, which is proportional\(^{10}\) to the mean across individuals of the squared individual-specific trend (all divided by twice mean income).

Write

\(^{10}\) Specifically, the variance of predicted values is the squared growth rate times the variance of the time index, where the time index $t$ is always defined so that it has mean zero, so that the constant term measures mean income. The variance of predicted values $r_i t$ for an individual $i$ is thus the square of the growth estimate $r_i$ times the variance of $t$ or $r_i^2 (T^2-1)/12$.  

6
\[ GE_2 = I + \left( D - \frac{1}{2} \sum_{i=1}^{L} \left( \frac{r_i^2 (r_i^2 - 1)}{12} \right) \right) + \frac{1}{2} \sum_{i=1}^{L} \left( \frac{r_i^2 (r_i^2 - 1)}{12} \right) \]

or

\[ GE_2 = I + (D - M) + M = I + V + M. \]

I will call the terms \( V \) for “volatility” and \( M \) for “mobility risk” though of course other measures of those concepts are also possible. \( V \) now captures squared deviations around the linear individual-specific trend in income. \( M \) measures the extent to which incomes grow or fall over time; it represents the expected squared trend in incomes. We can also additively decompose this into components proportional to the variance of trends and the expected trend squared. Write

\[ GE_2 = I + V + M = I + V + R + A. \]

where \( R \) is relative mobility risk and \( A \) is absolute mobility risk (proportional to the squared mean of estimated trends across all individuals, a constant). \( R \) measures how much incomes differentially grow or fall over time, or the dispersion of individual-specific trends in income; if \( T = 5 \), this measure is simply twice the variance of trends. If everyone experienced the same average income growth over time (i.e. \( r_i = E(r_i) \) for all \( i \)) then \( R = 0 \). Thus, \( R \) actually measures “relative mobility risk” or the variance of individual growth rates in income, not the mean growth rates (level of absolute mobility), nor the covariance of growth rates with mean levels of income (pro-rich growth).

However, the term \( A \) is negligible in applications presented here, since it is the squared mean across individuals of individual-specific growth rates in income divided by mean income squared and is therefore is very small relative to \( R \) (i.e., \( A \) is on the order of 2 to 5 percent of \( M \) in every instance examined, and would not be visible on a graph of trends over time).

Note that assuming linear growth in individual incomes and estimating the rates even over some short time period is not uncontroversial. Often researchers assume a constant percentage rate of growth in incomes over time, or regress log income on time. This assumption does not match the empirical distribution of income growth, and drops any observations with zero or negative income in a period (limiting the sample to those with lower variation over time since anyone with income that drops to or rises from that low level of income), but I duplicate all the estimates using log income for comparison purposes (results in the appendix).

We can embed the above calculations in a regression framework using panel data by writing a fixed-effects model with individual-specific linear time trends:

\[ y_{it} = u_i + r_i t + e_{it} \]

where \( u_i \) is an individual fixed effect, \( r_i \) is an individual growth rate, and \( e_{it} \) is the idiosyncratic error. We then estimate \( I \) by the variance of estimated fixed effects (with or without the adjustment for estimation error), \( V \) as the mean squared residual from the regression (plus the adjustment), and \( R \) as the variance of predicted values, measuring in
essence the variance of coefficients on $t$. These would all be divided by twice mean income in the sample to get $GE_2$ measures.

This regression framework suggests a further degrees-of-freedom adjustment implemented by scaling each quantity by $N/(N - 2L) = T/(T - 2)$ to account for the estimation of intercepts and trends ($2L$ parameters), but since this applies to each quantity, and does not affect their relative size in any way, I drop this adjustment. We might also adjust our estimate of the variance of individual growth rates $r_i$ using standard errors of those estimates, but it is not immediately clear what standard errors we should use, nor that the subtracting off the mean of squared estimated standard errors would be an improvement. In particular, if we tried to cluster on the panel identifier (essentially the approach taken in estimating standard errors for individual-specific means $u_i$) and estimate standard errors for individual growth rates $r_i$, we would get zero for each estimate.

The quantities as estimated are thus:

\[
\hat{I} = \left( B_i - \frac{W_i}{T - 1} \right),
\]

\[
\hat{\nu} = \left( \frac{T}{T - 1} \right) W_i - \frac{1}{2\hat{\nu}^2} \left[ L^{-1} \sum_{i=1}^{L} \left( \frac{r_i^2 (T_i^2 - 1)}{12} \right) \right], \text{ and}
\]

\[
\hat{M} = \frac{1}{2\hat{\nu}^2} \left[ L^{-1} \sum_{i=1}^{L} \left( \frac{r_i^2 (T_i^2 - 1)}{12} \right) \right],
\]

which together sum to the variance of income across all observations divided by twice mean income, which is defined as estimated total income risk:

\[
\hat{GE}_2 = \hat{I} + \hat{\nu} + \hat{M},
\]

so the sum is just the estimated $GE_2$ for all income observations, computed over both individuals and time.

Note that this measure does not characterize the progressivity of income growth or the change in income inequality over the period of $T$ years. Studying successive $T$-year periods, we can decompose changes in inequality, volatility, and mobility across periods. However, a straightforward set of measures of change within the $T$-year period involve the correlation of estimates of the mean level of income $u_i$ with volatility $\text{var}(e_{it}^2)$ and mobility $r_i$. To the extent that the individual mean level of income $u_i$ is correlated with the individual volatility $\text{var}(e_{it}^2)$ we can say that idiosyncratic risk is progressively distributed, suggesting that individuals may be making a risk-return tradeoff (or, possibly, much volatility may be regarded as positive shocks relative to baseline income). To the extent the mean level of income $u_i$ is correlated with mobility $r_i$, income growth can be said to be pro-rich, and in that case income inequality is rising during the $T$-year period. Heteroskedastic errors $e_{it}$ in the regression framework $y_{it} = u_i + r_i t + e_{it}$ with variance increasing in $t$ reflect increasing volatility patterns in the $T$-year period under study.
To measure $T$-year inequality, volatility, and mobility, we need only $T$ years of data on each individual in a survey (and to use weighted means instead of unweighted means in the previous formulas). For example, we can use five years from a longer panel and measure five-year inequality as the inequality across individuals in five-year averages of income. But this would say nothing about the trend in inequality.

If we want to measure trends in inequality, volatility, and mobility, we must of course have a much longer panel. Given a panel of some fixed length, for example $2T$ years of panel data, we can imagine computing a single $2T$-year measure of inequality and other components using a very small balanced panel (for only those individuals observed in every survey year), or using the first $T$ years to construct one estimate and the second nonoverlapping period of $T$ years (beginning in year $T + 1$ and running to year $2T$) to construct another estimate. Changes in $T$-year inequality, volatility, and mobility are then immediately apparent. More generally, with $2T$ years of data, we can construct $T + 1$ estimates, for each period of $T$ contiguous years.

**Data**

I use the Panel Study of Income Dynamics (PSID) data from the United States for survey years 1970 to 2005 (income years 1969 to 2004) to characterize trends in inequality, volatility, and mobility. Taxes are imputed using TAXSIM.11


A longer period $T$ is desirable for better estimates of the volatility component, but clearly if we wish to measure trends, a shorter period is preferable (so that we may compute more estimates across periods of length $T$). Note that $T$ must be at least 3 for each individual for that individual’s observations to be used, but that the variance of the idiosyncratic error term used to characterize volatility will tend to be dramatically understated for small $T$, and I will use $T = 5$ in this paper. I also compare results for $T = 5, 6, 7, 8,$ and 9 (results in the appendix) as a rough measure of sensitivity to this choice, indicating that longer periods increase volatility estimates and lower inequality estimates, as one might expect.

Another concern is that the variance of the idiosyncratic error term used to characterize volatility also captures measurement error, but this is in a deep sense inevitable—one cannot observe short-run variation in income and know whether it represents true short-run variation in income or misreported or mismeasured income. This

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applies even to administrative earnings records, or to datasets with merged administrative records and survey responses. The only approaches to separately estimate volatility and measurement error components require structural models of income distributions that can usually be rejected (in the statistical sense) by the very data used to fit them. To the extent that measurement error is increasing over time, any upward trend in volatility may represent increases in the volatility of true income, or increases in the volatility of measured income with no change in the volatility of true income.

I present results for family income for all individuals aged 30 to 60, including the cash value of transfers and cash-equivalent in-kind benefits (food stamps), including or excluding tax liabilities, with and without adjustment for family size. My adjustment for family size is accomplished by dividing family income by the square root of number of people in the family—see for example Coulter, Cowell, and Jenkins (1992) and Cowell and Jenkins (1995) on equivalence scales—but various alternative adjustments alter my results very little, possibly because I always restrict my sample to the nonelderly adult population and use individual person weights.

The degree of attrition over time in the PSID is substantial, hovering around 5 to 7 percent in most years. Over five years of data, spread over 11 calendar years, this can produce attrition rates of 30 percent or more when looking at a subset of data forming a balanced panel. To account for this type of attrition in short panels of length $T$ years across $2T – 1$ calendar years, I use the first year panel weights (adjusting for attrition from the survey up to that point) and calculate an adjustment factor to differentially adjust weights by $1/(1 – p)$ where $p$ is the estimated probability of attrition from a logit of attrition on characteristics (race, sex, and single year of age).

Topcoded income in the PSID represents a major threat to these decompositions, since the $GE_2$ index emphasizes variation in larger incomes (whereas a 90/10 ratio would be largely immune to this threat). For this reason, I drop the top two percent of income values in all years. I also compare these results (in the appendix) to fifty imputations of the top two percent of income values (imputing from a Pareto distribution with parameters estimated using income from the 90th percentile to the 98th), which should in some sense bound the size of the problem, and it does not appear to be a large issue.

Results

Looking first at five-year inequality, the impact of including taxes is apparent in both family-size-adjusted and unadjusted results (figures 1 and 2 respectively). After-tax incomes are substantially less unequal, with inequality index values about one third to two thirds as large as those for pretax incomes. This is to be expected, given the progressivity of the U.S. income tax system. The pattern over time of the ratio of pretax inequality to after-tax multiperiod inequality exhibits remarkable stability, ranging from 60 to 70 percent with a peak in 1991, and is remarkably similar using different accounting periods.

The effect of accounting for sampling variation in sample means of individual family incomes over five years of data is relatively small, lowering the estimate of inequality by about 7 percent (and simultaneously increasing volatility and mobility estimates by 25
percent). The effect on levels is visible in each graph, and the effect on measured trends is negligible.

The impacts of accounting period $T$ (shown in the appendix) are modest relative to the differences across pretax and after-tax estimates of inequality fixing the accounting period, though there is a clear ordering, where longer accounting periods produce lower estimates of long-run inequality. This is to be expected, as Shorrocks (1978) and other writers have noted, since there is some regression to the mean over time. The impact of family size adjustments on estimates of long-run inequality is quite modest compared to the effects of accounting for taxes or changing the period length. In fact, there is no difference visible in graphs until after 1996, and the indices differ only in the second significant digit in all years.

The impact of netting out estimated taxes from family income on estimated volatility, shown in figure 3, is similar to the impact on inequality. Volatility estimates are roughly 50 to 80 percent as large for after-tax income as for pretax income. Additionally, some of the year-to-year instability of estimates is substantially reduced after deducting tax liabilities from family income. The variance-stabilizing effects of income taxes are presumably due to the progressive rate structure in the US.

The impact of accounting period on estimated volatility is small, as shown in the appendix, but longer accounting periods lead to higher estimates of volatility. One reason for this is possibly downward bias in the estimator, so that longer accounting periods are more plausibly estimating dispersion in income around long-run trends, though the results of Breunig (2001) do not apply to this particular case. As the accounting period is extended from 5 to 9 years of data (9 to 17 calendar years), the impact of any further extensions of the accounting period grow smaller, which is consistent with the notion that as $T$ gets larger, the estimator is converging on some true value (though that notion would be misguided here, since the data used varies across estimates; the notion of asymptotic convergence would not apply in any application to real data). In any case, it is reassuring that the accounting period has only a small impact on volatility estimates.

The impact of family size adjustments on estimated volatility is reasonably small (comparing figure 4 to figure 3), raising the estimates by a factor of one third or so. This is largely due to the fact that family size tends not to adjust up and down over short time periods, but rather to evolve over a longer time period. That is, shocks due to divorce or marriage or the birth or departure of children tend to be persistent shocks. For that reason, much of the impact of family size adjustments is seen in the mobility risk estimates (comparing figure 6 to figure 5).

The impact of netting out taxes on mobility risk estimates (figure 5) is comparable to inequality and volatility, on the order of a one third reduction in estimated risk, also due to the progressivity of taxes. The family size adjustment produces more substantial differences for mobility, perhaps due to mobility in family size (changing household composition over the panel).

The impacts of accounting periods are much larger than for inequality or volatility estimates, as shown in the appendix. That accounting periods make more difference for mobility risk is not surprising, since much longer accounting periods offer more opportunity for low-probability sharp drops and rises to be observed, but more time over
which temporary shocks may be smoothed out by longer-term trends. If those who have declining income between two periods tend to have rising income the next, for regression to the mean in income growth rates, a longer accounting period would tend to lower estimated mobility risk, which is consistent with the observed effect in appendix figures 5 and 6. Another plausible story is that longer accounting periods tend to produce more precisely estimated coefficients for individuals, reducing the component of the variation due to estimation error (that component is not captured in the appendix figures).

Overall, these estimates suggest that long-run inequality increased about 50 percent, or increased by a factor of three halves, over the last 25 years. Similarly, volatility risk appears to have increased about 40 to 60 percent and mobility risk about 30 to 50 percent over the same period. Volatility and mobility risk estimates are less stable than long-run inequality estimates, in that they are more sensitive to specification choices, and long-run inequality is the dominant component of the aggregate income risk measure. The aggregate risk measure, summing inequality, volatility, and mobility risk, has also increased approximately 50 percent over the past 25 years.
Figure 1. Long-run inequality estimates (five years of data over 11 calendar years), for family income, with no adjustment for family size.
Figure 2. Long-run inequality estimates (5 years of data over 11 calendar years), for family income, dividing income by the square root of family size.
Figure 3. Year-to-year volatility estimates (5 years of data over 11 calendar years), for family income, with no adjustment for family size.
Figure 4. Year-to-year volatility estimates (5 years of data over 11 calendar years), for family income, dividing income by the square root of family size.
Figure 5. Multiyear mobility estimates (5 years of data over 11 calendar years), for family income with no adjustment for family size.
Regressivity of Growth

The regressivity of income growth within each five-year period can be measured as the correlation or covariance of mean income $u_i$ (mean over five years) and individual-specific growth rates $r_i$, as explained above. This is measured quite apart from the inequality, volatility, and mobility risk discussed in the previous section, though it is clearly related to trends in these measures; see Jenkins and Van Kerm (2006) for additional relevant discussion of pro-poor growth.

Figure 7 presents results for a period length $T$ of five years (other periods produce broadly similar results, as shown in the appendix), for both correlations and covariances,
imposing no adjustment for family size. In all years and all specifications, the regressivity measure is positive, indicating that richer individuals tend to enjoy greater income growth over the eleven-year periods studied. In the pretax income panels on the left, correlation is decreasing over time while covariance is increasing, which reflects the increasing variances that the covariance is divided by to obtain the correlation coefficient.

Using after-tax income as in the panels on the right, however, produces very different estimates. The trend in the regressivity of income growth when looking at pretax income is not clear, since one measure increases and the other falls. The regressivity of after-tax income, on the other hand, increased sharply through the 1980s and fell in the 1990s, measured either as a correlation or a covariance. These periods correspond respectively to a period following pro-rich tax reform in 1981, and a period following pro-poor tax reforms in 1990 and 1993. An identical pattern is observed using income deflated by the square root of family size (figure 8).
Figure 7. Regressivity of Income Growth, No Adjustment for Family Size.
Figure 8. Regressivity of Income Growth, Income Adjusted for Family Size.
Extensions

Straightforward extensions include decomposing the inequality, volatility, and mobility measures by income type (Shorrocks 1982) or by population subgroup (Shorrocks 1984). More generally, once we have embedded the measurement of inequality, volatility, and mobility in a panel regression framework as above, it is natural to consider measuring these quantities conditional on covariates. I.e. instead of writing

\[ y_{it} = u_i + r_t + e_{it} \]

we might write

\[ y_{it} = u_i + r_t + X_{it} \beta + e_{it} \]

and measure proportional reductions in inequality, volatility, and mobility due to some set of variables \( X \). This exercise is analogous to that of Cowell and Jenkins (1995). However, the interpretation of changes over time and the variability of the residual is substantially complicated by the introduction of covariates.

Alternatively, we might ask whether increases in inequality, volatility, and mobility are explained by changes in the distribution of \( X \) or changes in coefficients as in the Blinder-Oaxaca model (Jann 2008), or semi-parametrically estimate changes in inequality, volatility, and mobility not explained by changes in the distribution of \( X \) using the methods of Dinardo et al (1996) and others.\(^\text{12}\) Note this exercise can be done for changes across years, or across subpopulations within a year.

We might also replace our measure of income with another measure, for example after-tax income, and compare changes in inequality, volatility, and mobility and how they relate to changes in the tax code over time. Alternatives to this method include the decomposition into reranking (horizontal inequity) and ex post inequality (vertical inequity) proposed by King (1983) using an Atkinson index, or decomposition of a change (over time, but easily reconceived as two tax regimes instead of two calendar years) into progressivity of income changes and reranking proposed by Jenkins and Van Kerm (2006) using a generalized Gini index.

The proposed extensions offer the possibility of decomposing the effects of changes in education and labor markets, family structure, taxes, and other important factors on the observed changes in inequality, volatility, and mobility.

The regression framework could also be extended in another direction by moving to a multilevel mixed-effects regression framework, modeling \( r_i \) as a random effect, or adding a subscript \( i \) to \( \beta \) in the above equation (and perhaps assuming individual-specific trends are normally distributed). This type of extension trades a set of very strong and unverifiable assumptions for potentially improved efficiency of estimation. Nevertheless, moving in this direction offers the possibility of using large-sample theory to conduct tests on or construct confidence intervals for trends in inequality, volatility, and mobility, so the approach may appeal to the more parametrically minded.

\(^{12}\) See Dinardo (2002) and Lemieux (2002) for additional discussion of using reweighting to compare wage distributions.
Conclusions

The decomposition of variability in income across people and time undertaken here produces remarkably stable results across a variety of specifications. Calling total variability in incomes, measured as half the squared coefficient of variation or GE₂, a measure of income risk, it can be expressed as the sum of long-run inequality (a measure of income risk from behind the veil of ignorance\textsuperscript{13}), volatility or short-run fluctuations around a person-specific time trend, and variation in time trends or “mobility risk.” The results are relatively insensitive to the time period over which time trends are calculated, as summarized in figure 9, which compares results using five years or seven years of data for each individual, and shows each component “stacked” to indicate that they add up to total income risk. The results are also relatively insensitive to adjustments for family size, as shown in figure 10, comparing results using five years of data with no adjustment to those dividing family income by the square root of family size. All of these results indicate that long-run inequality is the dominant form of income risk in these data, but that all forms of income risk appear to be increasing over time.

\textsuperscript{13} On the veil of ignorance see Harsanyi (1953, 1955) and Rawls (1971).
Figure 9. Income Risk Decompositions by Accounting Period Length.

Figure 10. Income Risk Decompositions by Family Size Adjustment.
References


Appendices: Informal sensitivity analysis

A. Sensitivity to accounting period

The following figures reproduce figures from the text, using different period lengths $T$.

Figure A1. Long-run inequality estimates by accounting period, including estimated tax liabilities or not, with no adjustment for family size.
Figure A2. Long-run inequality estimates by accounting period, including estimated tax liabilities or not, dividing income by the square root of family size.
Figure A3. Year-to-year volatility estimates by accounting period, including estimated tax liabilities or not, with no adjustment for family size.
Figure A4. Year-to-year volatility estimates by accounting period, including estimated tax liabilities or not, dividing income by the square root of family size.

Pretax, Family-size-adjusted

After tax, Family-size-adjusted
Figure A5. Multiyear mobility estimates by accounting period, including estimated tax liabilities or not, with no adjustment for family size.
Figure A6. Multiyear mobility estimates by accounting period, including estimated tax liabilities or not, dividing income by the square root of family size.
Figure A7. Regressivity of income growth, no adjustment for family size.

Figure A8. Regressivity of income growth, income adjusted for family size.
B. Sensitivity to using imputing income for top two percentiles

Instead of dropping income values above the 98th percentile, which are very likely to be subject to differential topcoding across years, we might also impute those values from a Pareto distribution in each year, estimating the parameter alpha using incomes between the 90th and 98th percentile, and using the 90th percentile as the lower bound of the distribution. Estimated parameters (fitted via maximum likelihood) are shown in table B1, and results from 50 independent imputations are shown in figure B1.

Table B1. Pareto distributions used to impute the top two percent of income.

<table>
<thead>
<tr>
<th>Survey Year</th>
<th>98th %ile</th>
<th>90th %ile</th>
<th>alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>1978</td>
<td>105.33509</td>
<td>76.301659</td>
<td>8.5679449</td>
</tr>
<tr>
<td>1979</td>
<td>106.02862</td>
<td>76.354265</td>
<td>6.7376685</td>
</tr>
<tr>
<td>1980</td>
<td>102.60172</td>
<td>73.389345</td>
<td>7.7732366</td>
</tr>
<tr>
<td>1981</td>
<td>102.93658</td>
<td>71.956951</td>
<td>7.5134341</td>
</tr>
<tr>
<td>1982</td>
<td>103.20701</td>
<td>71.115054</td>
<td>7.4878477</td>
</tr>
<tr>
<td>1983</td>
<td>109.9004</td>
<td>74.885575</td>
<td>6.5834831</td>
</tr>
<tr>
<td>1984</td>
<td>112.43667</td>
<td>77.286356</td>
<td>7.9586754</td>
</tr>
<tr>
<td>1985</td>
<td>120.24363</td>
<td>79.339499</td>
<td>6.877939</td>
</tr>
<tr>
<td>1986</td>
<td>124.5292</td>
<td>82.444518</td>
<td>6.9192172</td>
</tr>
<tr>
<td>1987</td>
<td>146.60245</td>
<td>86.879949</td>
<td>5.9081968</td>
</tr>
<tr>
<td>1988</td>
<td>138.09871</td>
<td>87.835515</td>
<td>5.7427734</td>
</tr>
<tr>
<td>1989</td>
<td>148.88968</td>
<td>88.158087</td>
<td>5.835943</td>
</tr>
<tr>
<td>1990</td>
<td>156.95968</td>
<td>93.423111</td>
<td>6.3427248</td>
</tr>
<tr>
<td>1991</td>
<td>145.22603</td>
<td>90.040234</td>
<td>5.4257135</td>
</tr>
<tr>
<td>1992</td>
<td>147.6086</td>
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<tr>
<td>1993</td>
<td>148.68757</td>
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</tr>
<tr>
<td>1994</td>
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<tr>
<td>1995</td>
<td>163.88831</td>
<td>93.411556</td>
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<tr>
<td>1996</td>
<td>156.41409</td>
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<tr>
<td>1997</td>
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<td>95.127887</td>
<td>4.8283084</td>
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<tr>
<td>1999</td>
<td>175.61834</td>
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<tr>
<td>2001</td>
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<td>105.60792</td>
<td>4.0157911</td>
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<tr>
<td>2003</td>
<td>165.96979</td>
<td>100.12624</td>
<td>5.2739849</td>
</tr>
<tr>
<td>2005</td>
<td>183.05463</td>
<td>106.90015</td>
<td>5.0165794</td>
</tr>
</tbody>
</table>
As figure B1 shows, results are surprisingly insensitive to imputations from a Pareto distribution, though recent years appear to be more affected by imputations of the top two percent of the income distribution. It is likely the procedure adopted here of imputing year by year produces an upper bound for variance of volatility estimates (since several years for one individual are imputed with no dependence across years, artificially inflating the variance across years) and conversely a lower bound for variance of inequality estimates.
C. Sensitivity to using log family income instead of levels

The following figures reproduce figures from the text, using different period lengths T, and log family income instead of untransformed income.

Figure C1. Long-run inequality estimates by accounting period, including estimated tax liabilities or not, with no adjustment for family size.
Figure C2. Long-run inequality estimates by accounting period, including estimated tax liabilities or not, dividing income by the square root of family size.
Figure C3. Year-to-year volatility estimates by accounting period, including estimated tax liabilities or not, with no adjustment for family size.
Figure C4. Year-to-year volatility estimates by accounting period, including estimated tax liabilities or not, dividing income by the square root of family size.
Figure C5. Multiyear mobility estimates by accounting period, including estimated tax liabilities or not, with no adjustment for family size.
Figure C6. Multiyear mobility estimates by accounting period, including estimated tax liabilities or not, dividing income by the square root of family size.